STEAMTeach

Science | Technology | Engineering | Arts | Mathematics

MAKING TESSELLATIONS!

EDUCATIONAL TECHNOLOGY LAB, DEPT. OF EDUCATIONAL STUDIES, SCH. OF PHILOSOPHY, NATIONAL & KAPODISTRIAN UNIVERSITY OF ATHENS

Co-funded by the Erasmus+ Programme of the European Union

PROJECT

PROJECT ACRONYM	STEAMTeach
PROJECT TITLE	STEAM Education for Teaching Professionalism
PROJECT REFERENCE	2020-1-ES01-KA201-082102
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KEY ACTION	Cooperation for innovation and the exchange of good practices
ACTION TYPE	Strategic Partnerships for school education
PROJECT WEBSITE	https://www.steamteach.unican.es/

REPORT DETAILS

VERSION	1
DATE	20-1-22
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REVIEWED BY	-
STATUS	Final

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1 PROJECT JUSTIFICATION

This project can be an offspring for students to express themselves aesthetically, in a learning environment, through programming, where the making of Mathematical meaning can be the key to their personal expression. To make their own creations in 3D space through logo programming in MaLT2 (http://etl.ppp.uoa.gr/malt2/), a 3D Turtle Geometry environment with dynamic manipulation affordances, they should handle mathematical concepts and properties, making new meaning in a concrete or even abstract way.

1.1 Project Overview

Participant age:13-16	*No. of participants: a classroom*	*Duration: 3-4 school hours*
Level of knowledge: Preliminary knowledge of logo programming	*No. of teachers: 1-2*	*Type of venue: PC school lab*
Learning methodologies: PBL (project-based learning) Collaborative learning Constructionism	*Involved disciplines:* Technology Arts Mathematics	*Technological needs:* Computers Internet
Most emphasized learning methodology: PBL	*Main addressed topics:* Symmetry, tiling.	*Estimated project cost:* 0 €

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2 CURRICULAR CONTEXT

2.1 Key competences

The main key competences may be developed during the project development:

- Creativity
- Computational thinking
- Collaboration skills
- Taking initiative
- Critical thinking
- Originality

2.2 Content

The content from each discipline that will be addressed when implementing the project:

DISCIPLINE	CURRICULAR CONTENT ADDRESSED
Technology	The use of MaLT2, dynamic manipulation of 3D objects in the screen.
	Logo programming.
Engineering	Makers' culture.
	Design through experimentation.
Arts	Design and create mosaics, based on Escher's creations.
Mathematics	Geometrical figures.
	Symmetry (translation, rotation, reflection).



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The main aim is the tiling of a square frame with tiles, either using one kind of ready-made tile, or by creating a new tile, but not a square.

2.3 Expected learning outcomes.

What we expect students to achieve after their engagement with the project:

- Students make meaning on symmetry, use translation, rotation, and reflection to make figures.
- They use proportions and inverse proportions to make drawings.
- They use variables to express geometrical properties, in an algebraic way.

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3 STEPS TO BE EXECUTED

3.1 Step 1: Identifying the problem.

Duration: 0.5 SH (school hour).

The problem is the tiling of a square frame in the 3D space of MaLT2 with tiles, in a way that is aesthetically accepted and valuable for them (Figure 1). In this step, students should frame the problem, realize what they can do, and they cannot do in this learning environment, and identify its main characteristics, and constraints.

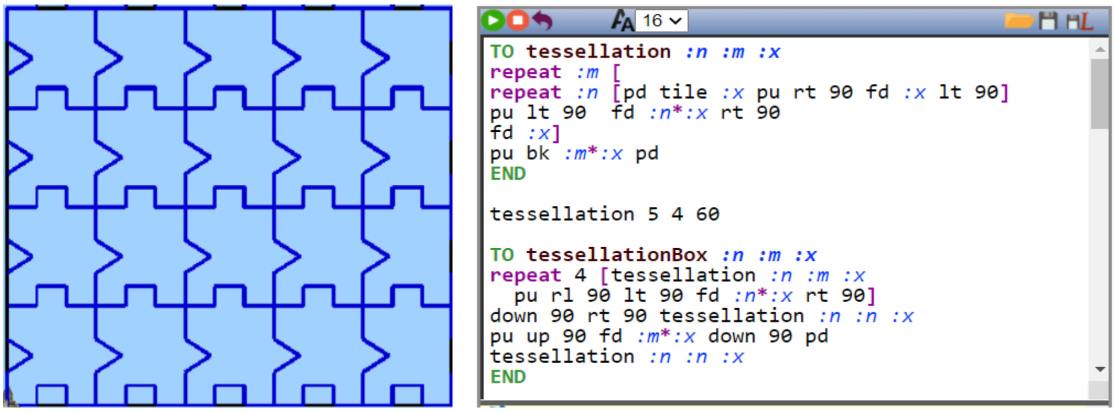

Figure 1: On the left-hand side: a case of tiling the square, in the environment of MaLT2. On the right-hand side: the logo editor containing the procedure that makes the tessellation.

3.1.1 Constraints

The only constraint is that the tile should not be a square, not only because the solution is trivial; the construction should be aesthetically valuable for the students.

It is obvious that the tiles should not overlap.

For the tiling, students may use some ready-made tiles that they are given to them as logo programs in MaLT2, or they could create their own tiles (through programming).

3.1.2 Criteria

The aesthetic result is the main criterion. We do not have to predetermine criteria of aesthetic quality, because we should let students express their understanding of it.

3.2 Step 2: Generating ideas.

Duration: 0.5 SH.

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The main objective of this step is that students introduce, discuss, and reflect on ideas about this type of tiling. Escher's paintings will be used as sparkers by the teacher, so that students recognize the three types of symmetry. The following themes could be used by the teacher as questions that will help students organize their activity:

- Brainstorming based on Escher's paintings and their relation to the project.
- Discussion and reflection on tiling based on Escher's paintings.
- Organizing the goals.
- Devising a strategy about how they will work.

As the inquiry will proceed, it is expected that the conversation and communication will be more Mathematically based.

Tip for teachers: If necessary, remind students of the criteria and constraints identified.

3.2.1 Sub-problems

Students could recognize some sub-problem, to make their own solutions.

- What types of symmetry does Escher use?
- How can we produce symmetry through programming?
- How can we avoid overlapping?

3.3 Step 3: Mathematical investigation.

Duration: 1 SH

In this step, students may try to answer the sub-problems above (or more of their own), giving provisional answers to start working. Some more specific questions to be answered are:

- How can we reproduce a tile in MaLT2 exploiting translation, rotation, or reflection?
- How can we use the given tiles to make tessellations and a mosaic in the square given?
- Will we use the given tiles, or are we going to create our own tile?

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- Are we going to use one type of symmetry or not? Will we choose this type from the beginning?

3.3.1 Experiment/task 1

The following figure (Figure 2) shows the tiling through rotation.

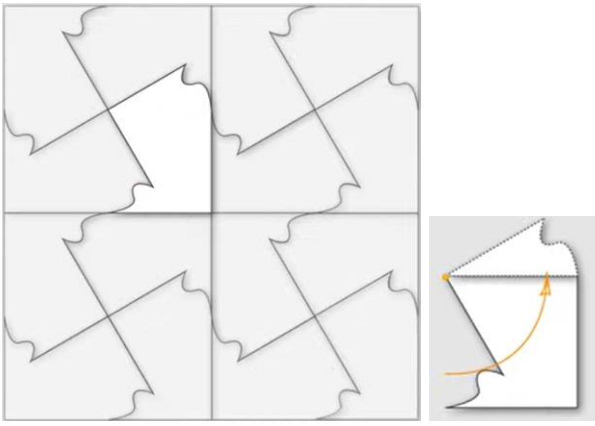

Figure 2: Experimenting with symmetry using paper and pencil.

Students may use paper and pencil to experiment with symmetry (rotation, translation, reflection) or use the program of the tile (below) and the slider on MaLT2 to change it (Figure 3) and reproduce it through translation or rotation.

```
to tile :x

setpencolor [0 0 255]
fd :x/3
right 60
fd :x/3
lt 120
fd :x/3
rt 60
fd :x/3
rt 90
fd :x/3
lt 90
fd :x/5
rt 90
fd :x/3
rt 90
fd :x/5
lt 90
fd :x/3
rt 90
```

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```
fd :x/3
left 60
fd :x/3
rt 120
fd :x/3
lt 60
fd :x/3
rt 90
fd :x/3
rt 90
fd :x/5
lt 90
fd :x/3
lt 90
fd :x/5
rt 90
fd :x/3
rt 90
end

tile 50
```

Figure 3: Using the slider, students may manipulate the value of x, changing the size of the tile.

3.3.2 Experiment/task 2

Formulate ideas through experimentation for tiling the square given with a tile of your preference and by using any of the symmetry's types?

In this task, students move forward trying to reproduce a tile, to make tessellations with no overlapping.

3.4 Step 4: Designing and Constructing the model.

Duration: 1 SH

During this step, students will generate many ways as possible to make tessellations of the square using MaLT2. They may exploit the experimentation of the previous phase, but they can construct a model based on a totally novel idea. The sub-problems they are supposed to address are:

- Implement their ideas from previous phases, to reproduce the tile in MaLT2.

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- Make the tiling fit into the square.
- Try different types of tiles.
- Try tiling using different types of symmetry.

Each group of students will suggest solutions for each sub-problem and will be a prototype of their project.

3.5 Step 5: Evaluating the model

Duration: 0.5 SH

The prototypes from the previous step will be discussed with the whole classroom and the teacher. They should be encouraged to exchange the prototypes and test them, to elaborate them after group discussion. The teachers may pose the following questions:

- Does it work?
- Does it address the problem?
- How do you like it?
- How could you improve the prototype?

3.6 Step 6: Refining the model.

Duration: 0.5 SH

After the group discussion, the exchange of the prototypes, and the reflection on them, student may refine their own prototype, based on the feedback they got.

At the end of the refining process, models could be presented to e-class.

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4 PROJECT EVALUATION

Proposing ways to evaluate the expected learning results.

To evaluate the learning process, we propose the exploitation of the learning outcomes of the lesson.

- Give the students a simple task asking them to apply symmetric transformations in a figure; assess their use of translation, rotation, and reflection.
- Ask students to write down the ratio of magnification or diminution of two given figures.
- Then ask them to make magnifications or diminutions of shapes, with given ratios.
- Give students tasks to express through formulas the relationship of lengths in given shapes.

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5 MATERIALS AND ROOMS

There is no cost since the school has a PC lab, and MaLT2 is free.

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6 INSTRUCTIONS ABOUT THE CONSTRUCTION OF PROTOTYPE/SOFTWARE/OTHER

An exemplary tile in logo is given above. You may create your own tiles. Here are some ready-made procedures in logo.

```
To tile1 :a
setpencolor [102 0 255]
fd :a/3 rt 60
fd :x/3 lt 120
fd :x/3 rt 60
fd :x/3 rt 90
fd :x/3 lt 90
fd :x/5 rt 90
fd :x/3 rt 90
fd :x/5 lt 90
fd :x/3 rt 90
fd :x/3 lt 60
fd :x/3 rt 120
fd :x/3 lt 60
fd :x/3 rt 90
fd :x/3 rt 90
fd :x/5 lt 90
fd :x/3 lt 90
fd :x/5 lt 90
fd :x/3 rt 90
```

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```
END


To tile2 :x
setpencolor [5 118 84]
rt 60 fd 2*:x/3
lt 120 fd 2*:x/3
rt 60 fd :x/3
rt 90 fd :x/3
lt 60 fd 2*:x/3
rt 120 fd 2*:x/3
rt 30 fd :α
rt 90 fd :x rt 90
END



To tile3 :x
setpencolor [255 123 0]
rt 60 fd 2*:x/3
lt 120 fd 2*:x/3
rt 60 fd :x/3
rt 90 fd :x
rt 30 fd 2*:x/3
rt 120 fd 2*:x/3
lt 60
fd :x/3
```

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```
rt 90 fd :x

rt 90

END



To frame

setpencolor [0 0 0]

repeat 2 [fd 200 rt 90 fd 250 rt 90]

END
```

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Papert, S., & Harel, I. (1991). *Situating Constructionism*. Ablex Publishing Corporation Norwood, NJ.

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