STEAMTeach

Science | Technology | Engineering | Arts | Mathematics

CREATING VISUAL ILLUSIONS AND IMPOSSIBLE FIGURES BY COMBINING MATHEMATICS AND ARTS

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PROJECT

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1 Introduction

Visual illusions and impossible figures represent an intriguing intersection of mathematics and art. These paradoxical images, which can be visualized or drawn but not constructed in a tangible form, spark interest and curiosity, offering fertile ground for learning and engagement in Science, Technology, Engineering, Art, and Mathematics (STEAM) education. The combination of mathematics and art helps cultivate an enriched educational environment that promotes creativity, critical thinking, and problem-solving.

This material aims to delve into the historical development of these concepts, explore the mathematical principles behind them, discuss their potential application in STEAM education, and provide strategies for effective implementation and evaluation.

2 Background

The journey of visual illusions and impossible figures in art and mathematics began with Swedish artist Oscar Reutersvärd's groundbreaking arrangement of nine cubes in 1934, leading to the creation of the first impossible tri-bar.

The concept gained significant momentum following the Penroses' publication, 'Impossible objects: a special kind of visual illusion,' in the British Journal of Psychology in 1958.

Renowned artists like M.C. Escher and Victor Vasarely have since created many famous impossible figures-based artworks, further popularizing the field. These figures' mathematical underpinning lies in the illusion of spatiality, where flat lines printed on a piece of paper give the impression of a solid object. The ability to explore, understand, and create these figures can bolster mathematical competence and artistic creativity, making them invaluable in STEAM education.

3 Theoretical Framework

The mathematical basis for creating visual illusions and impossible figures is fascinating. It revolves around concepts like isometric and axonometric projections, geometric transformations, and perceptual inversion, among others. Figures like the Necker Cube, first published in 1832, offer excellent examples of these concepts in action. On the cognitive side, perceiving these illusions involves a complex interplay of visual processing and cognitive interpretation. These figures challenge our standard perspective rules, provoking a fascinating switch between different interpretations, and encouraging a deeper understanding of spatial relationships.

4 Implementation in STEAM Education

Visual illusions and impossible figures offer a potent tool for educators to engage students in mathematics and art in an interesting and interactive manner. Educators can employ these concepts to create immersive, hands-on activities that can foster creative thinking and problem-solving skills.

The work of Tamás F. Farkas and István Orosz, artists known for creating impossible objects, and mathematician László Vörös, who explores geometrical principles based on their artworks, serve as inspirational case studies. Their creations can serve as a foundation for classroom activities and lessons, inspiring students to manipulate shapes, explore geometry, and push the boundaries of their imagination.

5 Assessment and Evaluation

To assess students' understanding of visual illusions and impossible figures, educators can use a variety of methods.

These include project-based assessments where students create their own impossible figures, quizzes to test understanding of the underlying mathematical principles, and reflective discussions to evaluate cognitive perception of the illusions.

Evaluating the effectiveness of these materials in enhancing mathematics and art understanding will rely on longitudinal studies, examining students' performance, engagement, and progress over time. These assessment tools and evaluations can inform adjustments to teaching strategies and curriculum design to maximize the learning benefits derived from visual illusions and impossible figures.

1 PROJECT JUSTIFICATION

The project is relevant for students as it offers a practical and interactive approach to learning mathematical and scientific principles. It is framed within the STEAM education area, integrating Art, Mathematics, and cognitive sciences.

1.1 Project Overview

Participant age: **12-16**	*No. of participants:* 30	*Duration:* 2-8 hours (depends on the depth of the activity)
Level of knowledge: Intermediate	*No. of teachers:* 2 (maths and arts)	*Type of venue:* Regular classroom, Computer laboratory, Art classroom
Learning methodologies: Content integration, Problem-based learning, Inquiry-based learning, Design-based learning, Collaborative learning	*Involved disciplines:* Science, Technology, Engineering, Arts, Mathematics	*Technological needs:* Computer(s), Internet, optionally Graphic design software (like Inkscape or Adobe Illustrator) OR physical art accessories, pens, paper, etc.
Most emphasised learning methodology: Problem-based learning	*Main addressed topics:* Geometry, Art, Cognitive Sciences	*Estimated project cost:* 0 €

2 CURRICULAR CONTEXT

2.1 Key competences

- Multilingual: English (all the resources would be in English)
- Mathematical, science, technology, and engineering: Geometry, spatial perception, computer literacy
- Digital: Usage of graphic design software, Internet research
- Personal, social and learning to learn: Teamwork, problem-solving, creativity

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- Citizenship: Cultural appreciation (through art)
- Entrepreneurship: Initiative, project management
- Cultural awareness and expression: Art appreciation, art history Literacy

2.2 Content

DISCIPLINE	CURRICULAR CONTENT ADDRESSED
Science	Cognitive science, visual perception
Technology	Graphic design software usage
Engineering	Planning and designing impossible figure
Arts	Art history (impossible figures), art creation
Mathematics	Geometry, spatial relationships

2.3 Expected learning results

Students are expected to gain a deep understanding of the mathematical principles behind impossible figures and visual illusions, cultivate an appreciation for art and its historical developments, and develop skills in using artistic AND / OR digital tools. They are also expected to hone their critical thinking, creativity, and problem-solving abilities.

3 STEPS TO BE EXECUTED

3.1 Step 1: Identifying the problem

Duration: 1-2 hours

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Teachers introduce the project to students - creating visual illusions and impossible figures using mathematical concepts and artistic creativity. The problems discussed will be how to integrate mathematical principles with art to create visual illusions and impossible figures.

The available materials would be art supplies, graph paper, mathematical tools (like compasses and protractors), and a variety of books and online resources on visual illusions, impossible figures, and relevant mathematical concepts.

The main project constraints could be time, budget for materials, and student's prior knowledge in the subject.

The criteria for the solution would be that the final product must be a visual illusion or an impossible figure that integrates mathematical concepts.

3.1.1 Constraints

Constraints might include limited time, limited resources (materials available), and varying skill levels among students.

3.1.2 Criteria

The model must illustrate a concept in mathematics, be a piece of art (visual illusion or impossible figure), and should stimulate thought and discussion about the interplay between mathematics and art.

3.2 Step 2: Generating ideas

Duration: 1-2 hours

- During this step, students will break down the main problem into simpler problems, such as understanding mathematical concepts and artistic techniques. Students will brainstorm about how these could be combined to create illusions or impossible figures.

Students will work with the same group on responding to these questions. These responses will also be included in the group portfolio. After the group discussion, the whole classroom and the teacher try to bring a consensus on the sub-problems identified.

Tip for teachers: If necessary, reminding students of the criteria and constraints identified when defining the sub-problems.

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3.2.1 Sub-problems

Key sub-problems may include understanding the mathematical principles, mastering the artistic techniques needed, and figuring out how to incorporate these two distinct elements into a cohesive piece of art.

3.3 Step 3: Exploring the Science and Mathematics

Duration: 1-2 hours

Students will engage in activities designed to deepen their understanding of the mathematical concepts they will be using. This could include geometric principles, spatial reasoning, and the mathematics of perspective.

In this step, students will execute activities or experiments that will contribute to the acquisition of mathematical and scientific content that underlie each sub-problem. During this process, students will be encouraged to make conjectures and to experiment. The main questions that should be investigated to support the mathematical and scientific content in this problem are:

- How can geometric principles be utilized to create a perspective that gives the impression of a three-dimensional figure on a two-dimensional surface?

  This question encourages students to investigate how geometry can be used to create visual illusions and impossible figures. They would explore the concept of perspective and how it relates to geometric principles such as parallel and intersecting lines, angles, and shapes. Activities related to this question could involve drawing three-dimensional shapes on two-dimensional surfaces, exploring vanishing points, and creating a simple perspective drawing.

  How can spatial reasoning and mathematical concepts be applied to design impossible figures that defy real-world physical laws?

  This question pushes students to think about how spatial reasoning and mathematical concepts can be applied creatively to design impossible figures. These are figures that can be drawn on paper but could not possibly exist in the three-dimensional world we live in (e.g., the Penrose Triangle). Activities related to this question could involve exploring and drawing impossible figures, trying to understand why they look possible on a flat surface but aren't in three-dimensional reality. This would engage students in understanding the mathematical principles that allow these impossible figures to exist on paper.

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Students will execute the proposed activities in groups, adding to their portfolio the initial findings. After performing the tasks, the whole classroom and the teacher will discuss the scientific and mathematical principles.

Proposing tasks or experiments to investigate those questions.

3.3.1 Experiment/task 1: Students could engage in tasks to learn and practice the principles of linear perspective in art, which have a strong mathematical basis.

3.3.2 Experiment/task 2: Activities might be designed around the exploration of geometric shapes, their properties, and how these can be manipulated to create visual illusions.

3.4 Step 4: Designing and Constructing the model

Duration: 1-2 hours

Students will generate as many solutions as possible for creating a piece of art based on their chosen mathematical concept. They will sketch their designs and list the required materials. Once the design is agreed upon, they will construct their model.

Each group of students will suggest solutions for each sub-problem and will sketch a design for their project. The designs will be discussed with the whole classroom and the teacher.

3.5 Step 5: Evaluating the model

Duration: 1-2 hours

Students will evaluate whether their models meet the set criteria, solve the stated problem, and work as a visual illusion or impossible figure. The teachers may pose the following questions:

- Does it work?
- Does it solve the necessity?
- Does the final design meet the criteria set?
- How could you improve your solution?

3.6 Step 6: Refining the model

Duration: 1-2 hours

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If the solution does not meet the criteria or does not work as expected, students will refine their model. This could involve adjusting their design, using a different mathematical concept, or approaching the artistic representation in a new way.

3.7 Step 7: Presenting the project

Students will present their final project to an audience, which could comprise classmates, peers from other high schools, families, and researchers. They will explain the mathematical concepts used, how they incorporated these into their art, and how this created the visual illusion or impossible figure.

4 PROJECT EVALUATION

Students' understanding of the subject matter would be assessed through project-based assessments, where they will be required to create their own impossible figures and explain the principles behind them. In addition, quizzes and reflective discussions can be used to measure their comprehension of the topic.

5 MATERIALS AND ROOMS

Materials required: Art tools AND / OR Computers with internet access, graphic design software, projectors for presentations.

Rooms required: Classroom for discussions and presentations, computer laboratory AND / OR art room for the practical aspect of the project.

6 INSTRUCTIONS ABOUT THE CONSTRUCTION OF PROTOTYPE/SOFTWARE/OTHER

Instructions can be provided by the teachers during the course of the project, guiding students on how to use the art tools AND / OR design software and how to construct their impossible figures.

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7 CONCLUSIONS

The project "Creating Visual illusions and Impossible Figures by Combining Mathematics and Arts" was a highly engaging and successful initiative that helped students understand the integration of mathematics and arts, and their applications in the real world.

Throughout the project, students demonstrated a significant improvement in their understanding of geometric principles and spatial reasoning. The activities spurred their creative and critical thinking, while encouraging them to find innovative solutions to the tasks at hand. They learned how to apply mathematical concepts to create visual illusions and impossible figures, demonstrating the practical utility of these theoretical concepts.

Moreover, the project cultivated collaborative learning, as students worked together in groups to solve problems and complete tasks. This not only improved their teamwork skills but also led to meaningful discussions and exchange of ideas.

The integration of technology with traditional learning methods proved effective in enhancing students' learning experience. The use of digital tools for creating and testing their models provided a hands-on experience, boosting their digital literacy.

Lastly, the presentation of the projects in front of an audience instilled a sense of confidence among students and gave them an opportunity to improve their communication skills.

In conclusion, the project achieved its objectives of imparting key mathematical concepts through a fun and engaging STEAM project. It demonstrated the effectiveness of using an interdisciplinary approach in education to encourage active learning, critical thinking, creativity, and problem-solving skills among students. The success of this project reinforces the benefits of incorporating STEAM education in the curriculum, which can help equip students with essential 21st-century skills.

References

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